

CYCLIC HEAT FLOW THROUGH AN INSULATED TRIANGLE EXPOSED TO THE LUNAR ENVIRONMENT

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May 1965

GPO PRICE \$ _____

CFSTI PRICE(S) \$ _____

Hard copy (HC) 2.00

Microfiche (MF) 50

653 July 65

N65-34402

(ACCESSION NUMBER)

38

(PAGES)

CR 67109

(NASA CR OR TMX OR AD NUMBER)

(THRU)

1

(CODE)

33

(CATEGORY)



RESEARCH LABORATORIES

BROWN ENGINEERING COMPANY, INC.

HUNTSVILLE, ALABAMA

TECHNICAL NOTE R-147

CYCLIC HEAT FLOW THROUGH AN INSULATED
TRIANGLE EXPOSED TO THE LUNAR ENVIRONMENT

May 1965

Prepared For

PROPULSION DIVISION
P&VE LABORATORY
GEORGE C. MARSHALL SPACE FLIGHT CENTER

By

RESEARCH LABORATORIES
BROWN ENGINEERING COMPANY, INC.

Contract No. NAS8-20073

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ABSTRACT

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This report describes a thermodynamic analysis, numerical analysis and FORTRAN computer program which analyzes the periodic heat flow through a section of a body into the interior of the body. The section is triangular in shape and one side is exposed to the lunar environment while the other side is exposed to the body's interior environment. It is assumed that this results in a constant heat transfer coefficient. A profile through the section consists of a thin outer metal skin, any thickness of high grade insulation, and a thin inner metal skin. The computer program is listed in Brown Engineering Program Library as Program No. SP-149.

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Approved

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LIST OF SYMBOLS

<u>Mathematical</u>	<u>FORTRAN</u>	<u>Definition</u>
A	A	Area of triangle
\bar{A}	ABAR	Magnitude of vector N_x , N_y , N_z
a		Time steps from start, t_0 , to any time, t
a_1		Time steps from start to time t_1
a_2		Time steps from start to time t_2
C	CD	Colongitude of sum at 00.00 G. M. T. in degrees
c	CP	Specific heat of insulation
$E_{1,j}$		Energy incident on unit area of outer surface of triangle at time point, j
$E_{m,j}$	EMIS	Emissive power of lunar surface at time point, j
e	E10T1	Emissivity of outer surface of triangle at temperature, $T_{1,j}$
\bar{f}	FBAR	Phase fraction (varies from 0 to 1)
G	G	Solar constant on lunar surface
H	HTC	Heat transfer coefficient from insulation to cabin
h	H	Magnitude of ξ steps
k	K	Magnitude of t steps
\bar{k}	KBAR	Thermal conductivity of insulation
L	LKH	Computational parameter (\bar{k}/H)
l	L	Thickness of insulation
l_n	XLN	x direction cosine of triangle outward normal

LIST OF SYMBOLS (Continued)

<u>Mathematical</u>	<u>FORTTRAN</u>	<u>Definition</u>
$l_{s,j}$	XLS	x direction cosine of a line from the triangle to the sun at time point, j
M1	M1	Number of points defining insulation in ξ direction ($M1 = m + 1$)
m	M	Number of space intervals through insulation (in ξ direction)
m_n	XMN	y direction cosine of triangle outward normal
$m_{s,j}$	XMS	y direction cosine of a line from the triangle to the sun at time point, j
N_x N_y N_z	$\left. \begin{array}{l} \text{XNX} \\ \text{XNY} \\ \text{XNZ} \end{array} \right\}$	x, y, z components of the vector product which defines the triangle outward normal
N1	N1	Number of points defining a lunation ($N1 = n + 1$)
n	N	Number of time intervals in one lunation
n_n	XNN	z direction cosine of triangle outward normal
$n_{s,j}$	XNS	z direction cosine of a line from the triangle to the sun at time point, j
p		Any period from the start of a lunation
q_A	QQ	Net heat flux passing through triangle into cabin from times t_1 to t_2
$\dot{q}_{A,j}$	QDOT	Heat flux passing into cabin over triangular area, A, at time point, j
$\dot{q}_{u,j}$		Heat flux passing into triangle per unit area at time, j
q_u		Net heat flow into cabin, per unit area, from times t_1 to t_2

LIST OF SYMBOLS (Continued)

<u>Mathematical</u>	<u>FORTRAN</u>	<u>Definition</u>
r	RMTS	Reflectivity of lunar surface to solar radiation
T_c	TC	Cabin temperature
$T_{i,j}$	T(I, J)	Temperature in insulation at distance $\xi = (i-1)h$ and time $(j-1)k$ after start of lunation
$T_{m,j}$	TM	Lunar surface temperature at time point, j
$T_{M1,j}$		Cabin wall temperature (i. e. , space point M1) at time point, j
$T_{1,j}$		Triangle outer surface temperature (i. e. , space point 1) at time point, j
$T_{\beta=0}$	TBET	Lunar equatorial temperature
t	TIME	Time, counted from 00. 00 G. M. T.
t_o		Period from 00. 00 G. M. T. to start of lunation
t_1, t_2		Arbitrary times, counted from 00. 00 G. M. T.
x, y, z		MOLAB coordinates, defined in Figure 2
$x^{(1)}, y^{(1)}, z^{(1)}$	XX1, YY1, ZZ1	Coordinates defining the triangle. Apexes are numbered 1, 2, 3 in a COUNTER-CLOCKWISE direction
$x^{(2)}, y^{(2)}, z^{(2)}$	XX2, YY2, ZZ2	
$x^{(3)}, y^{(3)}, z^{(3)}$	XX3, YY3, ZZ3	
α_m	E10TM	Absorptivity of triangle outer surface to lunar and solar radiation respectively
α_s	E10TS	
β	BETAD	Latitude of MOLAB in selenocentric coordinates (degrees)
	BETAR	Latitude of MOLAB in selenocentric coordinates (radians)

LIST OF SYMBOLS (Continued)

<u>Mathematical</u>	<u>FORTTRAN</u>	<u>Definition</u>
γ	GAM	Thermal diffusivity of insulation (defined by $k/\rho c$)
$\cos \epsilon_{1,j}$	COSALP	Cosine of the angle between triangle outward normal and a line to the sun at time point, j
θ	THED	Angular displacement of MOLAB x coordinate from local east (degrees)
	THER	Angular displacement of MOLAB x coordinate from local east (radians)
λ	LAMD	Longitude of MOLAB in selenocentric coordinates (degrees)
	LAMR	Longitude of MOLAB in selenocentric coordinates (radians)
$\lambda_{s,j}$	LAMS	Longitude of sun in selenocentric coordinates at time point, j
λ_0	LAMO	Longitude of sun in selenocentric coordinates at 00.00 G. M. T.
$\Delta \lambda_{s,j}$	DELLAM	Difference in longitude between MOLAB and sun at time point, j
ξ		Insulation space coordinate perpendicular to triangle. Origin at outside
ρ	RHO	Density of insulation
σ	SIG	Stefan-Boltzmann constant
τ	TAU	Period of one lunation

INTRODUCTION

As a result of the Apollo project, a need existed for a number of brief mathematical analyses. These were to be used to check quickly various aspects of design problems as they arose. This report provides a rapid means of checking one thermodynamic aspect of the Apollo project, namely, the heat fluxes passing into and out of a section of a body situated in the lunar environment.

The section which was analyzed was a triangular shape and, moving along a line perpendicular to the outer surface, was assumed to be comprised of the following elements: a thin metal outer skin; any thickness of high grade insulation; and a thin metal inner skin backing onto a constant temperature heat source or sink.

This particular shape was chosen because it is a relatively easy matter to split any body surface into triangular sections, and the elements of which it is assumed to be comprised are fairly representative of current practice.

The report consists of a thermodynamic analysis, a numerical analysis and a computer program written for an IBM 7094 computer which, subject to the restrictions outlined later, will accurately calculate the heat flux into the heat source or sink and the temperature distribution through the insulation.

ANALYSIS

Simplifications¹

1. The body surface is amenable to representation by triangles.
2. Each triangle can "see" no other part of the vehicle.
3. Conduction of heat between triangles is neglected².
4. Conduction of heat through the insulation has a trivial effect on an outer surface heat balance.
5. Periodic heat conduction through the insulation in one direction only is considered³.
6. The lunar surface is assumed to be an isothermal flat plane of infinite extent.
7. A single reflection of solar heat from the lunar surface is the only reflection of significance.
8. The insulation has a constant thermal conductivity.
9. A spherical moon is assumed to revolve around the sun in a circular orbit, coincident with the ecliptic, and to uniformly rotate with a period τ equal to a mean synodic month in a counterclockwise direction when viewed from the north.

Calculation of Triangle Outer Surface Temperature, $T_{1,j}$

Simplifications 3 and 4 indicate that⁴

$$E_{1,j} = e \sigma T_{1,j}^4 \quad (1)$$

where

$E_{1,j}$ - energy incident per unit area of triangle outer surface at time, j

e - emissivity of triangle at its own temperature, $T_{1,j}$

σ - Stefan-Boltzmann constant, and

$T_{1,j}$ - temperature of triangle outer surface.

In the light of simplifications 2 and 7 the energy incident per unit area of the triangle, $E_{1,j}$ comes from three sources: direct solar flux (represented by a constant solar constant, $G = 442 \text{ Btu/ft}^2 \text{ hr}$, see simplification 8); solar flux reflected once from the lunar surface; and direct lunar surface radiation (see simplification 6). Thus, if

$E_{m,j}$ - emissive power of lunar surface at time "j"

G - solar constant on lunar surface

r - reflectivity of lunar surface to solar radiation

n_n - "z" direction cosine of triangle outward normal

$n_{s,j}$ - "z" direction cosine of solar direction at time "j"

α_m - absorptivity of triangle to lunar radiation

α_s - absorptivity of triangle to solar radiation

$\epsilon_{1,j}$ - angle between triangle outward normal and solar direction at time "j", then

$$E_{1,j} = (\alpha_s G \cos \epsilon_{1,j})_i + \left[\frac{(1-n_n)}{2} \alpha_s r G n_{s,j} \right]_{ii} + \frac{(1-n_n)}{2} \alpha_m E_{m,j} \quad (2)$$

where $()_i = 0$ when $n_{s,j} \geq 0$ and $\cos \epsilon_{1,j} < 0$ (i. e., when the sun shines on the point λ, β but not on the triangle), and $()_i = ()_{ii} = 0$ when $n_{s,j} < 0$ (i. e., when the sun does not shine on the point λ, β). Thus, substituting Equation 2 into Equation 1 and rearranging, the outer surface temperature of the triangle (which is the outer surface temperature of the insulation, i. e., point 1) at time "j" is given by

$$T_{1,j} = \left\{ \frac{(1-n_n) [\alpha_m E_{m,j} + (\alpha_s r G n_{s,j})_i] + (2 \alpha_s G \cos \epsilon_{1,j})_{ii}}{2 e \sigma} \right\}^{\frac{1}{4}} \quad (3)$$

where $()_i = 0$ when $n_{s,j} \geq 0$ and $\cos \epsilon_{1,j} < 0$ and $()_i = ()_{ii} = 0$ when $n_{s,j} < 0$.

• Evaluation of Components of Unit Outward Normal

To evaluate the direction cosines of the outward normal from the triangle, it is first necessary to coordinate the triangle. This is done by numbering the apexes 1, 2, and 3 in a COUNTERCLOCKWISE direction when the triangle is viewed from OUTSIDE the body. Thus, the coordinates of the apexes in the coordinates x, y, and z shown in Figure 1 are $x^{(1)}$, $y^{(1)}$, $z^{(1)}$; $x^{(2)}$, $y^{(2)}$, $z^{(2)}$ and $x^{(3)}$, $y^{(3)}$, $z^{(3)}$.

A vector product of the vectors describing sides 1-2 and 1-3 results in the components of a normal

$$N_x = [y^{(2)} - y^{(1)}] [z^{(3)} - z^{(1)}] - [z^{(2)} - z^{(1)}] [y^{(3)} - y^{(1)}] \quad (4)$$

$$N_y = [z^{(2)} - z^{(1)}] [x^{(3)} - x^{(1)}] - [x^{(2)} - x^{(1)}] [z^{(3)} - z^{(1)}] \quad (5)$$

$$N_z = [x^{(2)} - x^{(1)}] [y^{(3)} - y^{(1)}] - [y^{(2)} - y^{(1)}] [x^{(3)} - x^{(1)}] \quad (6)$$

giving a resultant

$$\overline{A} = [(N_x)^2 + (N_y)^2 + (N_z)^2]^{\frac{1}{2}} \quad (7)$$

Note that the area of triangle, A, is given by

$$A = \frac{\overline{A}}{2} \quad (8)$$

Hence, the components of a unit normal (i. e., the direction cosines) in the x, y and z direction respectively, are given by

$$l_n = \frac{N_x}{\overline{A}} \quad (9)$$

$$m_n = \frac{N_y}{\overline{A}} \quad (10)$$

$$n_n = \frac{N_z}{\overline{A}} \quad (11)$$

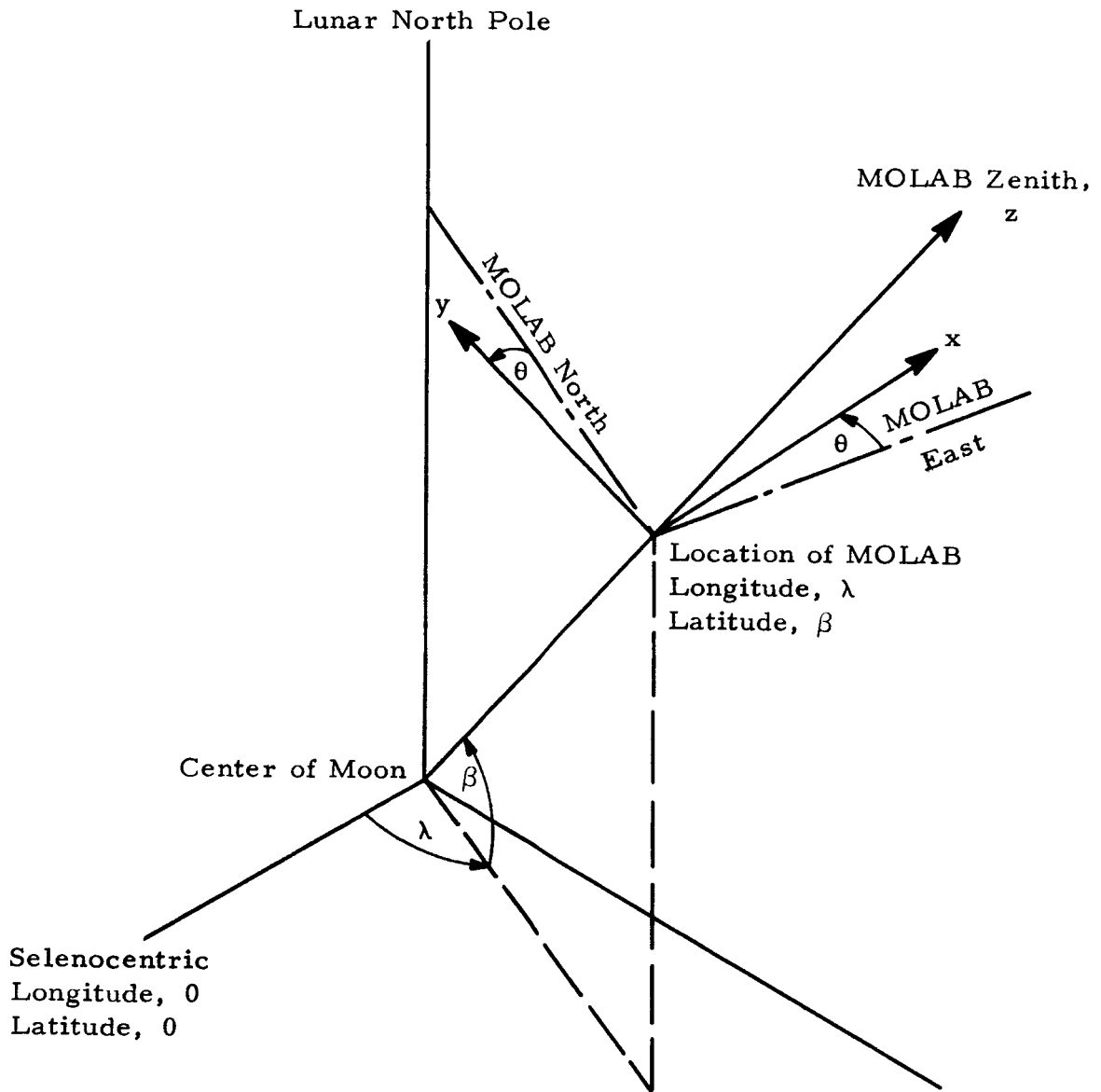


Figure 1. Orientation of Body Coordinates

• Evaluation of Time Dependent Terms; $n_{s,j}$, $\epsilon_{1,j}$, $E_{m,j}$

The longitude of the sun $\lambda_{s,j}$ at any time t in selenocentric coordinates is clearly given by

$$\lambda_{s,j} = \lambda_0 - \frac{t}{\tau} \quad (12)$$

where λ_0 is the longitude of the sun at $t = 0$. However, if the time t is counted from 00.00 G.M.T., then λ_0 is easily determined from Reference 5 as

$$\lambda_0 = 90 - C \quad (13)$$

where C = colongitude of the sun in selenocentric coordinates (tabulated in Reference 5). Thus, cognizance of Equations 12 and 13, together with the coordinate rotations shown in Figure 1, suggests the solar direction cosines $l_{s,j}$, $m_{s,j}$, $n_{s,j}$ in the x , y , z directions respectively may be represented by

$$l_{s,j} = -(\cos \theta \sin \Delta\lambda_{s,j} + \sin \beta \sin \theta \cos \Delta\lambda_{s,j}) \quad (14)$$

$$m_{s,j} = \sin \theta \sin \Delta\lambda_{s,j} - \sin \beta \cos \theta \cos \Delta\lambda_{s,j} \quad (15)$$

$$n_{s,j} = \cos \beta \cos \Delta\lambda_{s,j} \quad (16)$$

where

$$\Delta\lambda_{s,j} = \lambda - \lambda_{s,j} \quad (17)$$

The cosine of the angle between the sun and the unit normal, $\cos \epsilon_{1,j}$, is directly found from the scalar product of the unit sun vector and unit normal, thus

$$\cos \epsilon_{1,j} = l_{s,j} l_n + m_{s,j} m_n + n_{s,j} n_n \quad (18)$$

Frequently, the lunar surface temperature history is represented by a collection of points formed by the intersection of equatorial temperature against instantaneous phase fraction, $f = \Delta\lambda_{s,j}/2\pi$. (The points

recommended are shown in Appendix C.) The conversion from latitude $\beta = 0$ to any latitude β is accomplished by

$$T_{m,j} = T_{\beta=0} \cos^{1/6} \beta \quad . \quad (19)$$

(Equation 19 is sufficiently accurate for most purposes up to a latitude of about 45° .)

As the lunar surface acts very much like a black body, the emissive power $E_{m,j}$ is simply given by

$$E_{m,j} = \sigma T_{m,j}^4 \quad . \quad (20)$$

CALCULATION OF HEAT FLUX THROUGH THE INSULATION³

The object of the analysis is to determine the heat flux passing through a triangle into the cabin. Thus we must evaluate

$$q_A = \int_{t_1}^{t_2} \dot{q}_{A,j} dt = A \int_{t_1}^{t_2} \dot{q}_{u,j} dt \quad (21)$$

where

$$\dot{q}_{u,j} = H (T_{M1,j} - T_c) \quad (22)$$

However, as these expressions are the conclusions of the analysis below, more detailed considerations will be left until later.

The equation which describes the temperatures through the insulation is Fourier's Heat Conduction equation for one-dimension, which is

$$\frac{\partial^2 T}{\partial \xi^2} = \frac{1}{\gamma} \frac{\partial T}{\partial t} \quad (23)$$

where γ , the thermal diffusivity, is given by $\bar{k}/\rho c$.

Boundary Conditions

The exterior (i. e., triangle), $\xi = 0$

$$T = T(t) \quad (24)$$

the interior (i. e., cab), $\xi = \ell$

$$-k \left. \frac{\partial T}{\partial \xi} \right|_{\xi=\ell} = H (T_{M1,j} - T_c) \quad (25)$$

time bounds (cyclical array)

$$T(\xi, t) = T(\xi, t + \tau) \quad (26)$$

Numerical Procedure

Equation 23 with the boundary conditions represented by Equations 24, 25 and 26 indicate cyclical array of the form shown.

		One Cycle					time, t
Outer Surface	$T_{1,n}$	$T_{1,1}$	$T_{1,2}$. . .	$T_{1,n}$	$T_{1,N1} = T_{1,1}$	$T_{1,2}$
	$T_{2,n}$	$T_{2,1}$	$T_{2,2}$. . .	$T_{2,n}$	$T_{2,N1} = T_{2,1}$	$T_{2,2}$

	$T_{m,n}$	$T_{m,1}$	$T_{m,2}$. . .	$T_{m,n}$	$T_{m,N1} = T_{m,1}$	$T_{m,2}$
Inner Surface	$T_{M1,n}$	$T_{M1,1}$	$T_{M1,2}$. . .	$T_{M1,n}$	$T_{M1,N1} = T_{M1,1}$	$T_{M1,2}$
Distance ξ							

where the m uniform space intervals are of magnitude h , i. e. , $mh = l$ and the n uniform time intervals are of magnitude k , i. e. , $nk = \tau$ and the $T_{1,j}$ are known from repeated application of Equation 3. For convenience, the end points $m+1$ and $n+1$ were given the symbols $M1$ and $N1$ respectively.

Differences

Examination of the matrix indicates that the unknown temperatures are comprised of $T_{i,j}$; $i = 2$ to $M1$ and $j = 1$ to n , i. e. , $m \times n$ unknowns. These unknowns are solved by expressing Equations 23 and 25 in terms of the array temperatures, i. e. , by differencing Equations 23 and 25. Equation 25 may only be expressed by backward differences, and in the simplest form this gives

$$- \bar{k} \frac{(T_{M1,j} - T_{m,j})}{h} = H (T_{M1,j} - T_c) \quad (27)$$

$j = 1$ to n , i. e., n equations.

However, Equation 23 may be expressed in central difference form (usually the most accurate) and this yields

$$\frac{T_{i+1,j} + T_{i-1,j} - 2 T_{i,j}}{h^2} = \frac{1}{\gamma} \frac{(T_{i,j+1} - T_{i,j-1})}{2k} \quad (28)$$

$i = 2$ to m and $j = 1$ to n , i. e., $(m-1)n$ equations. Consequently, Equations 27 and 28 may now be applied at $n + (m-1)n$ points, i. e., $m \times n$ points. Thus, the problem is solvable.

Numerical Solution

To solve Equations 27 and 28 numerically, it is prudent to arrange them into a more convenient form thus

$$\left. \begin{array}{l} \text{for } i = 2 \text{ to } m \\ j = 1 \text{ to } n \end{array} \right\} T_{i,j} = \frac{1}{2} \left[T_{i+1,j} + T_{i-1,j} - \frac{h^2}{2\gamma k} (T_{i,j+1} - T_{i,j-1}) \right] \quad (29)$$

and for $j = 1$ to n

$$T_{M1,j} = \frac{1}{L+h} (h T_c + L T_{m,j}) \quad (30)$$

Application of the conventional iteration procedure then allows the equations to be solved numerically. However before the iteration procedure may be applied, a first guess is necessary. This guess was drawn up by assuming that the conduction through the insulation was linear with respect to distance which results in the following equation:

$$T_{i,j} = T_{1,j} - \frac{(T_{1,j} - T_c)}{l + L} \xi \quad (31)$$

The instantaneous heat flux per unit area, $\dot{q}_{u,j}$, is then immediately solved by application of

$$\dot{q}_{u,j} = H (T_{M1,j} - T_c) \quad . \quad (32)$$

Integration of the Heat Flux

As mentioned previously, the object is to calculate the heat flowing through a triangle between any two times t_1 and t_2 (which in practice are confined to multiples of the time step k , say $a_1 k$ and $a_2 k$ where $a_1 = (t_1 - t_0)/k$ and $a_2 = (t_2 - t_0)/k$)

$$q_A = A \int_{t_1}^{t_2} \dot{q}_{u,j} dt \quad . \quad (21)$$

However, this integral may be split into two parts thus

$$q_A = A \int_{t_0}^{t_2} \dot{q}_{u,j} dt - A \int_{t_0}^{t_1} \dot{q}_{u,j} dt \quad . \quad (32)$$

Hence if the integral $q_u = \int_{t_0}^t \dot{q}_{u,j} dt$ may be evaluated, then the problem

is solvable. The integral q_u itself may be reduced further because of the cyclical nature of the problem. Thus, if the integral q_u , $t_0 \leq t \leq t_0 + \tau$ and $(t - t_0)/a$ may be evaluated, then any other interval p greater than τ may be evaluated as follows:

$$\int_{t_0}^{t_0+p} = \int_{t_0}^{t_0+b\tau+\Delta} = b \int_{t_0}^{t_0+\tau} + \int_{t_0}^{t_0+\Delta}$$

where

$$b\tau + \Delta = p$$

$$0 \leq \Delta \leq \tau \quad .$$

The final evaluation of $q_A = A \int_{t_0}^{t_0 + t} \dot{q}_{u,j} dt$ was accomplished by the

trapezium rule thus

$$q_A = A \int_{t_0}^{t_0+t} \dot{q}_{u,j} dt = \frac{A}{2} \sum_{j=2}^a (\dot{q}_{u,j-1} + \dot{q}_{u,j}) k \quad . \quad (34)$$

FORTRAN PROGRAM

The program first reads all input data and then calculates the temperature distribution on the outer surface for one period. The program uses Equations 1 through 20 inclusively to calculate the temperature distribution. Next the initial guess for the temperature distribution is calculated from the outside wall temperatures using Equation 31.

At this point, the program begins an iterative procedure using Equations 29 and 30 to solve the matrix for the periodic temperature distribution. The program has reached the desired solution when the entire matrix is unchanged for two successive iterations.

The program next uses Equation 22 of the theory to calculate the heat flux into the cabin for all points in the period. And finally, the program calculates the total heat flux into the cabin, using Equation 34, and prints out the matrix of the temperature distribution.

CONCLUSIONS

The theory and program outlined in this report will calculate the periodic heat flux into the heat source or sink chosen and also the space time distribution of temperature through the insulation.

Extending the analysis to include temperature dependent thermal conductivities and temperature dependent heat transfer coefficients could easily be accomplished if required.

REFERENCES

1. Conway, L. and R. D. McGinnes, "Thermal Analysis of a Mobile Lunar Laboratory", Brown Engineering Company, Technical Note R-122, October 1964
2. Conway, L., "Temperature Analysis of an Arbitrarily Shaped Envelope Close to the Lunar Surface", Brown Engineering Company, Technical Note R-137, April 1965
3. Conway, L. and B. H. Kavanaugh, Jr., "Cyclic Heat Conduction Through Two-Layer Insulation", Brown Engineering Company, Technical Note R-146, May 1965
4. Kreith, F., "Radiation Heat Transfer", International Textbook Company, Scranton, Pennsylvania, 1962
5. "The American Ephemeris and Nautical Almanac", U.S. Government Printing Office, Washington, D. C., Published Yearly

APPENDIX A

PROGRAM INPUTS

The first 61 input cards are for a table of f versus $T_{\beta=0}$.
The format of these cards is 2F10.0. The first value is f and the second is $T_{\beta=0}$.

The next two cards contain

CARD 1

- | | | |
|----|-------------------------------|--------------------|
| 1. | The thickness of cab wall (L) | ft |
| 2. | The step size on ξ (H) | ft |
| 3. | The step size on t (K) | hr |
| 4. | The period (TAU) | hr |
| 5. | The cab temperature (TC) | $^{\circ}\text{R}$ |

CARD 2

- | | | |
|----|---------------------------------|--|
| 1. | Heat transfer coefficient (HTC) | $\text{Btu/ft}^2 \text{ hr } ^{\circ}\text{R}$ |
| 2. | The conductivity (KBAR) | $\text{Btu/ft hr } ^{\circ}\text{R}$ |
| 3. | The density (RHO) | lb/ft^3 |
| 4. | The specific heat (CP) | |

The format for these cards is 5F15.0

The next three cards contain

CARD 1

- | | | |
|----|----------------------------|-------|
| 1. | Lambda, degrees | LAMD |
| 2. | Beta, degrees | BETAD |
| 3. | C, degrees | CD |
| 4. | Theta, degrees | THED |
| 5. | G, Btu/hr ft^2 | G |
| 6. | e (dimensionless) | EIOT1 |
| 7. | α_m (dimensionless) | EIOTM |

CARD 2

- | | | |
|----|----------------------------|---------|
| 1. | α_s (dimensionless) | EIOTS |
| 2. | r (dimensionless) | RMTS |
| 3. | X coordinate of point 1 | XX1, in |
| 4. | Y coordinate of point 1 | YY1, in |

CARD 2 (Continued)

- | | | |
|----|-------------------------|---------|
| 5. | Z coordinate of point 1 | ZZ1, in |
| 6. | X coordinate of point 2 | XX2, in |
| 7. | Y coordinate of point 2 | YY2, in |

CARD 3

- | | | |
|----|-------------------------|---------|
| 1. | Z coordinate of point 2 | ZZ2, in |
| 2. | X coordinate of point 3 | XX3, in |
| 3. | Y coordinate of point 3 | YY3, in |
| 4. | Z coordinate of point 3 | ZZ3, in |

The format for these cards is 7F10.0.

The next card contains

- | | | |
|----|---|-------|
| 1. | Sigma, Btu/ft ² hr °R ⁴ | (SIG) |
| 2. | Number of steps on X plus 1 | (M1) |
| 3. | Number of steps on t plus 1 | (N1) |

The format of this card is E14.8, 2I5.

The next card contains

- | | | |
|----|---|-------|
| 1. | The starting point for calculation
of QQ | (MM1) |
| 2. | The stopping point for calculation
of QQ | (MM2) |

APPENDIX B
LIST OF FORTRAN PROGRAM

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```
C    DIMENSION T(21,1001)
      DIMENSION T(21,101)
      DIMENSION FAR(61),TBET(61)
      REAL L,K,KBAR,LKH
      REAL LAMR,LAMD,LAMO,LAMS
      PI = 3.1415926536
      DO 70 II = 1,61
70    READ (5,666)FAR(II),TBET(II)
666  FORMAT(2F10.0)
      READ(5,14)NCASE
14   FORMAT(I5)
      DO 11 KASE=1,NCASE
C    READ(5,10)L,H,K,TAU,TC,HTC, KBAR,RHO,CP
      READ(5,10)L,H,K,TAU,TC,HTC, KBAR,RHO,CP,TOL
10   FORMAT(5F15.0)
      READ (5,676)LAMD,BETAD,CD,THED,      G,EIOT1,EIOTM,EIOTS,RMTS,XX1,
1YY1,ZZ1,XX2,YY2,ZZ2,XX3,YY3,ZZ3
```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```

676 FORMAT(7F10.0      )
      READ(5,677)SIG,M1,N1
677 FORMAT(E14.8,2I5)
      READ(5,88)MM1,MM2
88  FORMAT(2I5)
      LKH=KBAR/HTC
      GAM = KBAR/RHO/CP
      N = TAU/K
      M = L/H
C    WRITE(6,275)N,M,N1,M1
      WRITE(6,275)N,M,N1,M1 ,KBAR,TOL
C 275 FORMAT(1H1,10X,3HN =,I6,4X,3HM =,I6,4X,4HN1 =,I6,4X,4HM1 =,I6////)
275  FORMAT(1H1,10X,3HN =,I6,4X,3HM =,I6,4X,4HN1 =,I6,4X,4HM1 =,I6,
      15X,4HKBAR,E18.8,3X,3HTOL,E18.8)
      RAD = PI/180.0
      LAMR = LAMD*RAD
      BETAR = BETAD*RAD
      CRAD=CD*RAD
      LAMO = (PI/2.0)-CRAD
      THER = THED*RAD

```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```
XX1=XX1/12.0
XX2=XX2/12.0
XX3=XX3/12.0
YY1=YY1 /12.0
YY2=YY2 /12.0
YY3=YY3 /12.0
ZZ1=ZZ1 /12.0
ZZ2=ZZ2 /12.0
ZZ3=ZZ3 /12.0
XNX = (YY2-YY1)*(ZZ3-ZZ1) - (YY3-YY1)*(ZZ2-ZZ1)
XNY = (ZZ2-ZZ1)*(XX3-XX1)-(ZZ3-ZZ1)*(XX2-XX1)
XNZ = (XX2-XX1)*(YY3-YY1) - (XX3-XX1)*(YY2-YY1)
ABAR = SQRT(XNX*XNX + XNY*XNY + XNZ*XNZ)
A = 0.5*ABAR
XLN = XNX/ABAR
XMN = XNY/ABAR
XNN = XNZ/ABAR
WRITE(6,101)RAD,LAMR,BETAR,CRAD,LAMO,THER,XX1,XX2,XX3,YY1,YY2,YY3,
1ZZ1,ZZ2,ZZ3,XNX,XNY,XNZ,ABAR,A,XLN,XMN,XNN
101 FORMAT(1H , 5X,6E20.8)
```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```
C
C      INPUT OR CALCULATE T(M1,N1) NOW
C
      DO 300 J = 1,N1
      XJ = J-1
      LAMS = LAMO - 2.0*PI*XJ*K/TAU
      DELLAM= LAMR-LAMS
      XLS = -(COS(THER)*SIN(DELLAM)+SIN(BETAR)*SIN(THER)*COS(DELLAM))
      XMS = SIN(THER)*SIN(DELLAM) - SIN(BETAR)*COS(THER)*COS(DELLAM)
      XNS = COS(BETAR)*COS(DELLAM)
      COSALP = XLS*XLN + XMS*XMN + XNS*XNN
      FBAR=10.0+DELLAM/(2.0*PI)
      IFB = FBAR
      FB1 = IFB
      FBAR = FBAR - FB1
C
C      BEGIN INTERPOLATION ROUTINE HERE
C
      DO 71 KT = 1,60
      KT 1 = KT
```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```
IF(FBAR.EQ.FAR(KT1)) GO TO 72
```

```
KT2 = KT + 1
```

```
IF(FBAR.GT.FAR(KT1).AND.FBAR.LT.FAR(KT2))GO TO 73
```

```
71 CONTINUE
```

```
WRITE(6,665)
```

```
665 FORMAT(1H1,15X,27HINTERPOLATION NOT POSSIBLE /1H1)
```

```
72 TINT = TBET(KT1)
```

```
GO TO 77
```

```
73 DIFF0 = FAR(KT2)-FAR(KT1)
```

```
DIFF1 = FBAR - FAR(KT1)
```

```
DIFT = TBET(KT2)-TBET(KT1)
```

```
DIFT1 = DIFT*DIFF1/DIFF0
```

```
TINT = TBET(KT1) + DIFT1
```

```
77 CONTINUE
```

```
C
```

```
C      END OF INTERPOLATION ROUTINE
```

```
C
```

```
TMBAR = TINT*SQRT(SQRT(SQRT(COS(BETAR))))
```

```
TM = 1.8*TMBAR
```

```
EMIS=SIG*TM**4
```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```

C1 = EIOTS*G* $\cos\alpha$ P
C2 = EIOTS*RMTS * G * XNS
C3 = EIOTM*EMIS
C4 = 1.0/(EIOT1*SIG)
IF(XNS.GE.0.0.AND. $\cos\alpha$ P.LT.0.0)C1 = 0.0
IF(XNS.GT.0.0)GO TO 700
C1 = 0.0
C2 = 0.0
700 CONTINUE
TEMP = ((1.0-XNN)*(C3+C2)/2.0)+C1
T(1,J)=SQRT(SQRT(C4*TEMP))
300 CONTINUE
WRITE(6,101)(T(1,J),J=1,N1)
DO 3000 I = 2,M1
XX = I-1
XX = XX*H
DO 3000 J = 1,N1
TX1 = T(1,J)
TX2 = 1.0 -HTC*XX/(HTC*L+KBAR)
TX3 = HTC *TC*XX/(HTC*L+KBAR)

```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

$T(I,J)=TX1*TX2+TX3$

3000 CONTINUE

C

C END OF INPUT OR CALCULATION OF T(M1,N1)

C

IT = 1

XIPT = M1

IPT = (XIPT/6.0)+1.0

IPT = 56/(2+IPT)

IXIT=(M1-1)*(N1-1)

IIXT = N1-1

64 ICT=0

IICT = 0

DO 21 J = 2,N1

J1M = J-1

J1P = J+1

DO 21 I = 2,M1

I1P = I+1

I1M = I-1

IF(I.EQ.M1)GO TO 30

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```

      IF(J.EQ.N1)J1P=2
      TIJ = 0.5*(T(I1P,J)+T(I1M,J)-H*H/(2.0*K*GAM)*
      1(T(I,J1P)-T(I,J1M)))
      GO TO 22
30 I1 = I-1
      TIJ    = (TC*H + T(I1,J)*LKH)/(LKH+H)
C  22 IF(ABS(TIJ-T(I,J)).LE.0.0001)ICT = ICT+1
      22 IF(ABS(TIJ-T(I,J)).LE.TOL    )ICT = ICT+1
C      IF(I.EQ.M1.AND.ABS(TIJ-T(I,J)).LE. 0.0001)IICT=IICT+1
      IF(I.EQ.M1.AND.ABS(TIJ-T(I,J)).LE. TOL    )IICT=IICT+1
      T(I,J)=TIJ
      IF(J.EQ.N1)T(I,1)=T(I,J)
21 CONTINUE
      WRITE(6,3037)IT,IICT,IIXT,ICT,IXIT
3037 FORMAT(1H ,25X,5I10)
1010 CONTINUE
      IF(ICT.EQ.IXIT)GO TO 1012
      IT=IT+1
      GO TO 64
1012 CONTINUE

```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```
      DIMENSION QDOT(6)
      WRITE(6,99)IT
99  FORMAT(1H1,50X,4HIT =,I7)
      WRITE(6,89)
89  FORMAT(1H0,50X,19HTHESE ARE THE QDOTS)
      I = 0
      DO 81 J = 1,N1
      I = I +1
      QDOT(I) = HTC*A*(T(M1,J)-TC)
      IF(I.NE.6.AND.J.NE.N1)GO TO 81
      WRITE(6,101)(QDOT(II),II=1,I)
      I = 0
81  CONTINUE
      WRITE (6,99)IT
      IPRT = 0
      J = 1
      WRITE (6,100)J
100 FORMAT(1H0,50X,3HJ = ,I7)
      WRITE (6,101)(T(I,J),I=1,M1)
      IPRT = IPRT+1
```

LIST OF FORTRAN PROGRAM

FOR MOLAB STUDY

```

AKHTC2 = A*K*HTC/2.0
QQ=0.0
MM2 = MM2-1
DO 82 J = MM1,MM2
  J1=J+1
C   IF(J.EQ.MM2)J1=MM1+1
C 82 QQ = T(M1,J)+T(M1,J1) - 2.0*TC + QQ
C   QQ = A*K      • HTC/2.0*QQ
      QQ=(T(M1,J)+T(M1,J1)-2.0*TC)*AKHTC2 + QQ
      WRITE (6,100)J1
      WRITE(6,101)(T(I,J1),I=1,M1),QQ
      IPRT=IPRT+1
      IF(MOD(IPRT,IPT).EQ.0)WRITE(6,99)IT
82 CONTINUE
      WRITE(6,87)QQ
C 87 FORMAT(1H1/////55X,4HQQ =,E20.8//////////50X,
C 122H***** END OF JOB *****/1H1)
87 FORMAT(1H1/////55X,4HQQ =,E20.8//////////50X,
123H***** END OF CASE *****//)
11 CONTINUE
STOP
END

```

APPENDIX C

LUNAR SURFACE EQUATORIAL TEMPERATURE HISTORY

Phase Fraction	Lunar Surface Equatorial Temperature (°K)
(Noon) .000	390
.016	389
.030	387
.050	383.5
.060	380.0
.083	376.0
.100	371.0
.1016	361.0
.1030	353.0
.1500	341.0
.1600	330.0
.1830	313.0
.2000	292.0
.2160	266.0
.2300	227.0
.2500	145.0
.2600	119.0
.2830	116.0
.3000	112.0
.3160	110.0
.3300	109.0
.3500	105.0
.3600	103.0
.3830	101.0
.4000	100.0
.4160	99.0
.4300	98.0
.4500	97.0
.4600	96.0
.4830	95.0
(Midnight) .5000	94.0
.5160	93.0
.5300	92.0
.5500	92.0
.5600	92.0
.5830	91.0
.6000	91.0
.6160	91.0
.6300	90.0

Phase Fraction	Lunar Surface Equatorial Temperature (°K)
.6500	90.0
.6600	89.0
.6830	89.0
.7000	88.0
.7160	88.0
.7300	87.0
.7500	87.0
.7600	214.0
.7830	259.0
.8000	291.0
.8160	312.0
.8300	330.0
.8500	342.0
.8600	353.0
.8830	361.0
.9000	370.0
.9160	377.0
.9300	380.0
.9500	385.0
.9600	388.0
.9830	390.0
(Following Noon) 1.0000	390.0

DOCUMENT CONTROL DATA - R&D*(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)*

1. ORIGINATING ACTIVITY (Corporate author) Research Laboratories Brown Engineering Company, Inc. Huntsville, Alabama		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP N/A	
3. REPORT TITLE "Cyclic Heat Flow Through an Insulated Triangle Exposed to the Lunar Environment"			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Technical Note, May 1965			
5. AUTHOR(S) (Last name, first name, initial) Conway, Dr. Lawrence and Kroupa, Thomas J., III			
6. REPORT DATE May 1965		7a. TOTAL NO. OF PAGES 38	7b. NO. OF REFS 5
8a. CONTRACT OR GRANT NO. NAS8-20073		9a. ORIGINATOR'S REPORT NUMBER(S) TN R-147	
b. PROJECT NO. N/A		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) None	
c.			
d.			
10. AVAILABILITY/LIMITATION NOTICES None			
11. SUPPLEMENTARY NOTES None		12. SPONSORING MILITARY ACTIVITY Marshall Space Flight Center NASA	
13. ABSTRACT <p>This report describes a thermodynamic analysis, numerical analysis and FORTRAN computer program which analyzes the periodic heat flow through a section of a body into the interior of the body. The section is triangular in shape and one side is exposed to the lunar environment while the other side is exposed to the body's interior environment. It is assumed that this results in a constant heat transfer coefficient. A profile through the section consists of a thin outer metal skin, any thickness of high grade insulation, and a thin inner metal skin. The computer program is listed in Brown Engineering Program Library as Program No. SP-149.</p>			14. KEY WORDS computer program thermodynamic analysis heat transfer insulation lunar environment triangular body section